Year 11-12 transition pack
A-Level Maths
Due Thursday 1st September 2022

Name: ________________

You must complete all of the questions.
Make sure you understand why you are performing each calculation
Answers to applied are at the end for you to mark your work.
Your work will be reviewed by your teachers on arrival into Year 12
You will be given an assessment on this content during the first few lessons

For the first parts of this booklet you will need to remember that you can simplify powers of the same base using the laws of indices:

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(ab)^n = a^n b^n$

(1) Indices, roots, and surds (Numerical - Non-calculator)

Warm up

1. Find the value of:

a) $9^{\frac{1}{2}}$ b) $9^{-\frac{1}{2}}$ c) $\left(\frac{1}{4}\right)^{\frac{1}{2}}$ d) $\left(\frac{1}{4}\right)^{-\frac{1}{2}}$ e) $\left(\frac{125}{64}\right)^{\frac{1}{3}}$

2. Rationalise the denominator of:

a) $\frac{1}{\sqrt{3}}$ b) $\frac{2}{\sqrt{3}}$ c) $\frac{3}{\sqrt{3}}$ d) $\frac{\sqrt{2}}{\sqrt{3}}$ e) $\frac{3\sqrt{2}}{\sqrt{3}}$

3. Write the following in the form $a\sqrt{b}$ where a is an integer or a fraction, and b is an integer

a) $27^{\frac{1}{2}}$ b) $27^{-\frac{1}{2}}$ c) $\left(\frac{1}{8}\right)^{\frac{1}{2}}$ d) $\left(\frac{1}{8}\right)^{-\frac{1}{2}}$ e) $\left(\frac{125}{64}\right)^{\frac{1}{2}}$

Examples

Find the value of:

$5\left(\frac{343}{125}\right)^{\frac{1}{3}} = 5\left(\frac{\sqrt[3]{343}}{\sqrt[3]{125}}\right) = 5\left(\frac{7}{5}\right)$

$= \frac{5\times 7}{5}$ (you can divide both numerator and denominator by 5)

$= 7$

$\frac{1}{3}\left(\frac{1000}{729}\right)^{-\frac{2}{3}} = \frac{1}{3}\left(\frac{729}{1000}\right)^{\frac{2}{3}} = \frac{1}{3}\left(\frac{\sqrt[3]{729}}{\sqrt[3]{1000}}\right)^2$

$= \frac{1}{3}\left(\frac{9}{10}\right)^2$

$= \frac{1}{3}\left(\frac{81}{100}\right) = \frac{1\times 81}{3\times 100} = \frac{27}{100}$

Write in the form $a\sqrt{b}$:

$5(10)^{-\frac{1}{2}} = 5\left(\frac{1}{10^{1/2}}\right) = 5\left(\frac{1}{\sqrt{10}}\right) = \frac{5}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}}$

$= \frac{5\sqrt{10}}{10} \begin{matrix}\div 5\\ \div 5\end{matrix}$

$= \frac{\sqrt{10}}{2}$

$13 = 12 + 1$

$11^{\frac{13}{3}} = 11^{\frac{12+1}{3}} = 11^{4+\frac{1}{3}} = 11^4 \times 11^{1/3}$

$= 14621\sqrt[3]{11}$

$11 \times 11 = 121$; 121×121: $121 + 2420 + 12100 = 14641$

$5^{-\frac{1}{2}} + \sqrt{125} = \frac{1}{\sqrt{5}} + \sqrt{25\times 5}$

$= \frac{1}{\sqrt{5}}\frac{\times\sqrt{5}}{\times\sqrt{5}} + 5\sqrt{5}$ factorise $\sqrt{5}$

$= \frac{\sqrt{5}}{5} + 5\sqrt{5} = \sqrt{5}\left(\frac{1}{5}+5\right) = \sqrt{5}\left(\frac{26}{5}\right)$

$= \frac{26}{5}\sqrt{5}$

$\left(4\left(27^{-\frac{1}{2}}\right) + 27^{\frac{1}{2}}\right) - \left(4\left(3^{-\frac{1}{2}}\right) + 3^{\frac{1}{2}}\right)$

$\left[4\left(\frac{1}{\sqrt{27}}\right) + \sqrt{27}\right] - \left[4\left(\frac{1}{\sqrt{3}}\right) + \sqrt{3}\right]$

$\sqrt{27} = \sqrt{9}\times\sqrt{3} = 3\sqrt{3}$

$\left[\frac{4}{3\sqrt{3}}\frac{\times\sqrt{3}}{\times\sqrt{3}} + 3\sqrt{3}\right] - \left[\frac{4}{\sqrt{3}}\frac{\times\sqrt{3}}{\times\sqrt{3}} + \sqrt{3}\right]$

expand the -1

$\left[\frac{4\sqrt{3}}{9} + 3\sqrt{3}\right] - 1\left[\frac{4\sqrt{3}}{3} + \sqrt{3}\right] = \frac{4\sqrt{3}}{9} + 3\sqrt{3} - \frac{4\sqrt{3}}{3} - \sqrt{3}$

$= \sqrt{3}\left(\frac{4}{9} + 3 - \frac{4}{3} - 1\right) = \frac{10}{9}\sqrt{3}$

Your turn

Find the value of:

$10\left(\frac{27}{1000}\right)^{\frac{1}{3}}$

$\frac{1}{5}\left(\frac{343}{1000}\right)^{-\frac{2}{3}}$

Write in the form $a\sqrt{b}$:

$15(3)^{-\frac{1}{2}}$

$17^{\frac{16}{5}}$

$7^{-\frac{1}{2}} + 3\sqrt{7}$

$\left(10\left(8^{-\frac{1}{2}}\right) + 8^{\frac{1}{2}}\right) - \left(10\left(2^{-\frac{1}{2}}\right) + 2^{\frac{1}{2}}\right)$

Warm up

1. Write the following in index notation (e.g., $3x^2$, or $\frac{4}{3}x^{-\frac{3}{2}}$ are in index notation)

a) $\sqrt[3]{x}$ b) $\frac{1}{x^2}$ c) $\frac{1}{\sqrt{x}}$ d) $\sqrt{a^4}$ e) $\frac{\sqrt{t^7}}{t^3}$

2. Write the following in index notation (e.g., $3x^2 + \frac{4}{3}x^{-\frac{3}{2}}$ is in index notation)

a) $a^5 + \frac{1}{a^2}$ b) $3y^5 - \frac{4}{t^3}$ c) $\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}$

d) $\frac{3}{x} - \frac{2}{y} + \sqrt{x}$ e) $\sqrt[3]{x} + \frac{2}{y^2}$

Examples

Write the following in index notation:

$\frac{5}{10\sqrt{x}} = \frac{5}{10}\times\frac{1}{\sqrt{x}} = \frac{1}{2}\times\frac{1}{x^{1/2}} = \frac{1}{2}\times x^{-1/2} = \frac{1}{2}x^{-1/2}$

$\sqrt{x} = x^{1/2}$ $\frac{1}{x^{1/2}} = x^{-1/2}$

$\frac{10\sqrt[3]{y}}{5y^2} = \frac{10}{5}\times\frac{\sqrt[3]{y}}{y^2} = \frac{1}{2}\times\frac{y^{1/3}}{y^2} = \frac{1}{2}\times y^{1/3-2} = \frac{1}{2}y^{-5/3}$

$\frac{3\sqrt[3]{x}+1-x^2}{9x} = \frac{3x^{1/3}+1-x^2}{9x} = \frac{3x^{1/3}}{9x}+\frac{1}{9x}-\frac{x^2}{9x} = \frac{3}{9}\times\frac{x^{1/3}}{x}+\frac{1}{9}\times\frac{1}{x}-\frac{1}{9}\times\frac{x^2}{x} = \frac{1}{3}x^{-2/3}+\frac{1}{9}x^{-1}-\frac{1}{9}x$

$x\sqrt{x}+\frac{1}{x^2}(\sqrt[3]{x}) = x\times x^{1/2} + x^{-2}\times x^{1/3} = x^{1+1/2}+x^{-2+1/3} = x^{3/2}+x^{-5/3}$

$\frac{8x^{\frac{1}{2}}\times 24x^{\frac{3}{2}}}{16x} = \frac{8\times x^{1/2}\times 24\times x^{3/2}}{16\times x} = \frac{8\times 24\times x^{1/2}\times x^{3/2}}{16\times x} = \frac{8\times 24}{16}\times\frac{x^{1/2}\times x^{3/2}}{x} = \frac{24}{2}\times\frac{x^2}{x} = 12\times x = 12x$

$\frac{x^{\frac{2}{3}}}{\frac{2}{3}}+\frac{x^{-\frac{1}{3}}}{-\frac{1}{3}} = x^{2/3}\div\frac{2}{3}+x^{-1/3}\div -\frac{1}{3} = x^{2/3}\times\frac{3}{2}+x^{-1/3}\times -\frac{3}{1} = \frac{3}{2}x^{2/3}-3x^{-1/3}$

Your turn

Write the following in index notation:

$\frac{4}{20x}$

$\frac{64y^2}{8\sqrt[3]{y}}$

$\frac{100\sqrt{x}+20x^4+2}{5x^3}$

$x\sqrt[3]{x}+\frac{1}{x^3}(x^2)$

$\frac{40x^{\frac{1}{2}}\times 5x^{-\frac{5}{2}}}{1000x}$

$\frac{x^{\frac{1}{3}}}{\frac{1}{3}}+\frac{x^{-\frac{2}{3}}}{-\frac{2}{3}}$

Warm up

1. Write the following expressions in index form without brackets or fractional indices

a) $(3x)^2$ b) $(3+x)^2$ c) $(25x)^{\frac{1}{2}}$ d) $(4x)^2 - 5x^2$ e) $(4x)^{\frac{1}{2}} - 5\sqrt{x}$

2. Expand, and then write the following in index notation (e.g., $3x^2 + \frac{4}{3}x^{-\frac{3}{2}}$ is in index notation)

a) $(x^2+x)^2$ b) $\left(x+\sqrt{x}\right)^2$ c) $\left(x+\frac{1}{x}\right)^2$ d) $\left(3x+\frac{2}{x}\right)^2$ e) $\left(3x^2+\frac{2}{x^3}\right)^2$

Examples

Expand, and then write the following in index notation as a sum of three different terms

$$\frac{\left(3\sqrt{x}+2\right)^2}{\sqrt{x^3}} = \frac{(3\sqrt{x}+2)(3\sqrt{x}+2)}{x^{1/3}}$$

$$= \frac{9x + 6\sqrt{x} + 6\sqrt{x} + 4}{x^{1/3}}$$

$\boxed{\sqrt{x} = x^{1/2}}$

$$= \frac{9x^1}{x^{1/3}} + \frac{12x^{1/2}}{x^{1/3}} + \frac{4}{x^{1/3}} = 9x^{2/3} + 12x^{1/6} + 4x^{-1/3}$$

Write the following expressions without using brackets or fractional and negative indices

$3x^{-1} = \frac{3}{x}$ ($3 \times \frac{1}{x}$) $(3x)^{-1} = \frac{1}{3x}$ $\frac{1}{3}x^{-1} = \frac{1}{3x}$ ($\frac{1}{3} \times \frac{1}{x}$) $\left(\frac{1}{3}x\right)^{-1} = \left(\frac{x}{3}\right)^{-1} = \frac{3}{x}$

$3x^{-2} = \frac{3}{x^2}$ ($3 \times \frac{1}{x^2}$) $(3x)^{-2} = \frac{1}{9x^2}$ ($\frac{1}{(3x)^2}$) $(3x)^2 = 9x^2$ $\left(\frac{1}{3}x\right)^{-2} = \left(\frac{x}{3}\right)^{-2} = \frac{9}{x^2}$

$3x^{\frac{1}{2}} = 3\sqrt{x}$ $(3x)^{\frac{1}{2}} = \sqrt{3x}$ $3x^{-\frac{1}{2}} = \frac{3}{\sqrt{x}}$ ($3 \times \frac{1}{\sqrt{x}}$) $(3x)^{-\frac{1}{2}} = \frac{1}{\sqrt{3x}}$

$\frac{1}{2}x^{\frac{1}{2}} = \frac{1}{2}\sqrt{x}$ $\left(\frac{1}{2}x\right)^{\frac{1}{2}} = \frac{\sqrt{x}}{\sqrt{2}}$ ($\left(\frac{x}{2}\right)^{\frac{1}{2}}$) $\frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$ ($\frac{1}{2} \times \frac{1}{\sqrt{x}}$) $\left(\frac{1}{2}x\right)^{-\frac{1}{2}} = \left(\frac{x}{2}\right)^{-\frac{1}{2}} = \frac{\sqrt{2}}{\sqrt{x}}$

$\frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$ $\left(\frac{3}{2}x\right)^{\frac{1}{2}} = \frac{\sqrt{3x}}{\sqrt{2}}$ or $\frac{\sqrt{6x}}{2}$ $\frac{3}{2}x^{-\frac{1}{2}} = \frac{3}{2\sqrt{x}}$ $\left(\frac{3}{2}x\right)^{-\frac{1}{2}} = \left(\frac{3x}{2}\right)^{-\frac{1}{2}} = \frac{\sqrt{2}}{\sqrt{3x}}$

$(125x)^{\frac{1}{3}} + 125x^{\frac{1}{3}} + (125x)^{-\frac{1}{3}}$

$= \sqrt[3]{125x} + 125\sqrt[3]{x} + \frac{1}{\sqrt{125x}} = 5\sqrt[3]{x} + 125\sqrt[3]{x} + \frac{1}{5\sqrt[3]{x}}$

$= 130\sqrt[3]{x} + \frac{1}{5\sqrt[3]{x}}$

$\left(\frac{1}{4}x\right)^2 + \left(\frac{2}{9}x\right)^{-1} - (5x)^{-1}$

$\left(\frac{x}{4}\right)^2 + \left(\frac{2x}{9}\right)^{-1} - \frac{1}{5x} = \frac{x^2}{16} + \frac{9}{2x} - \frac{1}{5x}$

$= \frac{x^2}{16} + \frac{1}{x}\left(\frac{9}{2} - \frac{1}{5}\right) = \frac{x^2}{16} + \frac{43}{10x}$

$\left(\frac{2}{5x}\right)^{-1} = \frac{5x}{2}$

Your turn

Expand, and then write the following in index notation as a sum of three different terms

$$\frac{\left(\sqrt{x}+2x\right)^2}{\sqrt{x}}$$

Write the following expressions without using brackets or fractional and negative indices

$10x^{-1}$ $(10x)^{-1}$ $\frac{1}{10}x^{-1}$ $\left(\frac{1}{10}x\right)^{-1}$

$10x^{-2}$ $(10x)^{-2}$ $(10x)^2$ $\left(\frac{1}{10}x\right)^{-2}$

$10x^{\frac{1}{2}}$ $(10x)^{\frac{1}{2}}$ $10x^{-\frac{1}{2}}$ $(10x)^{-\frac{1}{2}}$

$\frac{1}{10}x^{\frac{1}{2}}$ $\left(\frac{1}{10}x\right)^{\frac{1}{2}}$ $\frac{1}{10}x^{-\frac{1}{2}}$ $\left(\frac{1}{10}x\right)^{-\frac{1}{2}}$

$\frac{7}{10}x^{\frac{1}{2}}$ $\left(\frac{7}{10}x\right)^{\frac{1}{2}}$ $\frac{7}{10}x^{-\frac{1}{2}}$ $\left(\frac{7}{10}x\right)^{-\frac{1}{2}}$

$100x^{\frac{1}{2}} + (100x)^{\frac{1}{2}} + (100x)^{-\frac{1}{2}}$

$\left(\frac{1}{4}x\right)^{-2} + \left(\frac{5}{7}x\right)^{-1} + (49x)^{\frac{1}{2}}$

$\left(\frac{3}{2x}\right)^{-1}$

(4) Simultaneous equations

Warm up

Solve the following simultaneous equations

$$x + y = 4$$

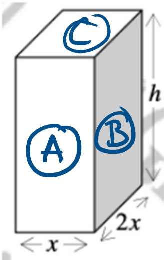
$$4y^2 - x^2 = 12$$

(4 marks)

Example

The figure shows the design of a fruit juice carton with capacity of $1000cm^3$. The design of the carton is that of a closed cuboid whose base measures x cm by 2x cm, and its height is h cm.

Show that the area of the juice carton, A cm^2, is given by



area A : $x \times h = xh$

area B : $2x \times h = 2xh$

area C : $x \times 2x = 2x^2$

① $\boxed{A} = 4x^2 + \frac{3000}{x}$

$$A = A \times 2 + B \times 2 + C \times 2$$

$$= (xh) \times 2 + (2xh) \times 2 + (2x^2) \times 2$$

$$= 2xh + 4xh + 4x^2$$

② $\boxed{A = 6xh + 4x^2}$

However, there is no h in formula ①, but we know the capacity (volume) of the carton is $1000cm^3$

$V =$ width × height × depth

$1000 = x \times h \times 2x$ make h the subject.

$\boxed{1000 = 2x^2 h}$

$$h = \frac{1000}{2x^2}$$

$$h = \frac{500}{x^2} \quad \text{or} \quad 500x^{-2}$$

② : $A = 6x\left(\frac{500}{x^2}\right) + 4x^2$

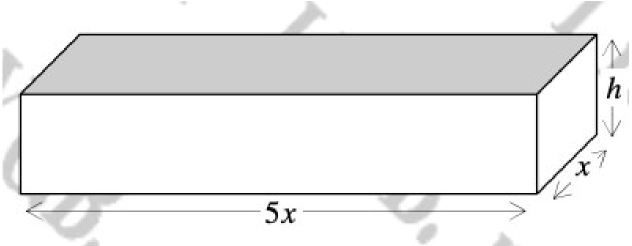
$$= \frac{3000x}{x^2} + 4x^2$$

$$= \frac{3000}{x} + 4x^2$$

$A = 4x^2 + \frac{3000}{x}$ as required

Your turn

1.



The figure above shows a **solid** brick, in the shape of a cuboid measuring $5x\ cm$ by $x\ cm$ by $y\ cm$.
The total surface area of the brick is 720 cm^2.

Show that the volume of the brick, V cm^3, is given by

$$V = 300x - \frac{25}{6}x^3.$$